FLUX DIFFUSION UPON THE COMPRESSION OF A MAGNETIC FIELD BY FLAT STRIPS OF VARIABLE WIDTH

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An equation is obtained describing the flux diffusion in flat profiled generators having magnetic cumulation (MC). The critical modes of operation of such generators on active and inductive loads are calculated.

1. The rapid compression of a magnetic field in a closed conducting circuit – magnetic cumulation – is used to obtain powerful pulsed currents and superstrong magnetic fields. Among the various types of generators having magnetic cumulation the flat generators [1, 2], which consist of two flat strips closed on a load (Fig. 1), are distinguished by simplicity of construction. In operation such a generator is connected to a source of current, which is pumped in during a time t_0, and then the strips are coupled with one another, and the magnetic flux is squeezed out from the cavity of the generator into the load.

In an ideal generator without resistance the flux is conserved, and the current pulse in the load is described by the equation

$$I(t) = I_0 L_0 / L(t) \tag{1.1}$$

where L_0 is the initial inductance of the generator, I_0 is the initial current in it, and $L(t)$ is the inductance of the generator in the course of operation.

In practice the operation of generators having magnetic cumulation is accompanied by flux losses which affect the size and shape of the current pulse. The flux losses can be divided into three parts: diffusion of the magnetic field into the strips of the generator, flux losses in the effective resistance of the load, and flux losses in the coupling of the strips. The latter effect is connected with the capture of flux in the closed cavities formed by the irregular surfaces of the conductors. It does not seem possible to take this factor into account, and in this connection we will henceforth consider the diffusion of the magnetic field into the strips and the "leakage" of flux in the resistance of the load.

Flat generators having magnetic cumulation are used mainly to obtain powerful current pulses. The magnetic fields in the cavities of such generators do not exceed $5 \cdot 10^5$ G, and therefore heating of the conductor and variation in its conductivity are slight. The load consists of an induction coil of rectangular cross section of length l_1 and width $2Z_0$. The distance between the strips is constant and equal to a. The conductance of the strips is assumed to be constant; the resistance R of the load is also constant. The length of the generator is l_0, and the width 2Z of the strips varies along the length. Neglecting the flux losses upon the coupling of the strips, one can assume that the compression of the magnetic field is accomplished by a perfectly conducting piston moving with a constant velocity D.

When the strips are sufficiently wide ($Z \gg \alpha$) the field in the load can be taken as uniform and equal to $B_1(t_1)$. The current in the generator is $I = cZ_0B_1(t_1)/2\pi$, and the field in the cavity is $B_2(y_1, t_1) = B_1(t_1)Z_0/Z(y_1)$. The flux in the generator cavity and the load has the form

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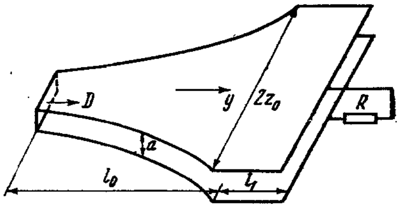

Fig. 1

$$F_1(t_1) = B_1(t_1)\,al_1 + B_1(t_1) \int\limits_{-l_0+Dt}^{0} \frac{aZ_0}{Z(y_1)}\,dy_1 = \frac{Z}{2\pi} B_1(t_1)\,L_1(t_1) \tag{1.2}$$

where $L_1(t_1)$ is the inductance of the system, which is a function of time

$$L_1(t_1) = \frac{2\pi}{Z_0}\,al_1 + 2\pi \int\limits_{-l_0+Dt}^{0} \frac{a\,dy_1}{Z_1(y_1)} \tag{1.3}$$

2. The equation for the flux in a flat generator having magnetic cumulation can be obtained from the equation of the electromagnetic induction for a circuit Γ traced along the boundary of the cavity and the load:

$$-\frac{1}{c}\frac{dF_1}{dt_1} = \oint\limits_{\Gamma} \mathbf{E}\,d\mathbf{y}_1 + RI \tag{2.1}$$

From the quasisteady system of equations for the field in the stationary strips and from Ohm's law we have

$$\mathbf{E} = \frac{1}{\sigma}\mathbf{j} = \frac{c}{4\pi\sigma}\,\mathrm{rot}\,\mathbf{B}_1 \tag{2.2}$$

Here $B_1(x_1, y_1, t_1)$ is the field in the conductor. Since the thickness of the skin layer is much less than the characteristic size in which the width of the strips changes significantly, the derivatives of B_1 with respect to x_1 make the main contribution to the electric field, and the equation of electromagnetic induction can be written in the form

$$\frac{dF_1}{dt_1} = \frac{2c^2}{4\pi\sigma}\frac{\partial B_1}{\partial x_1}\Big|_{x_1=0} \left(l_1 + \int\limits_{-l_0+Dt}^{0} \frac{Z_0}{Z_1(y_1)}\,dy_1\right) - \frac{c^2RZ_0}{2\pi} B_1(t_1) \tag{2.3}$$

or in the dimensionless variables

$$\begin{gathered} x_1 = ax,\quad t_1 = l_0t/D,\quad y_1 = l_0y,\quad Z_1 = Z_0Z,\quad l_1 = l_0l,\quad L_1 = L_1(0)L, \\ B_1 = B_0B,\quad F = BL,\quad F_1 = Z_0B_0L_1(0)F/2\pi \\ \frac{d(BL)}{dt} = \frac{2}{\mu}L(t)\frac{\partial B}{\partial x}\Big|_{x=0} - \frac{1}{\tau}B(t) \end{gathered} \tag{2.4}$$

The parameter $\mu = 4\pi\sigma a^2D/c^2l$ represents the magnetic Reynolds number; $\tau = L_1(0)D/c^2Rl_0$ is the relaxation time of the magnetic flux in the resistance R normalized to the time of operation of the generator.

The reduced inductance

$$L(t) = \frac{L_1(t)}{L_1(0)} = \frac{1}{\lambda}\left(1 + l^{-1}\int\limits_{-1+t}^{0}\frac{dy}{Z(y)}\right) \tag{2.5}$$

is expressed through the tuning coefficient $\lambda = L_1(0)/L_1(1)$ of the circuit and the ratio $l = l_1/l_0$ of the length of the load to the length of the generator. The derivative $(\partial B/\partial x)_{x=0}$ at the boundary of a stationary conductor can be expressed through the field B(t) at the boundary of the conductor and the initial distribution $B_0(x)$ of the field in the conductor [3]:

$$\frac{\partial B}{\partial x}\Big|_{x=0} = -\sqrt{\frac{\mu}{\pi}}\frac{d}{dt}\int\limits_0^t \frac{B(\theta)\,d\theta}{\sqrt{t-\theta}} + \sqrt{\frac{\mu}{\pi}}\frac{1}{\sqrt{t}} + \frac{2}{\sqrt{\pi}}\int\limits_0^\infty B_0'\left(\xi\sqrt{\frac{4t}{\mu}}\right)e^{-\xi^2}d\xi \tag{2.6}$$

and after substitution into (2.4) one obtains the equation for the field in the load of a profiled flat generator having magnetic cumulation:

$$\frac{d}{dt}(BL) = -\frac{2}{\sqrt{\pi\mu}}L(t)\frac{d}{dt}\int\limits_0^t\frac{B(\theta)\,d\theta}{\sqrt{t-\theta}} + \frac{2}{\sqrt{\pi\mu}}\frac{L(t)}{\sqrt{t}} + \frac{4}{\sqrt{\pi}}\frac{L(t)}{\mu}\int\limits_0^\infty B_0'\left(\xi\sqrt{\frac{4t}{\mu}}\right)e^{-\xi^2}d\xi - \frac{1}{\tau}B(t) \tag{2.7}$$

Equation (2.7) allows one to program the rise of the field in the load, i.e., by inserting a given monotonically increasing function B(t) one can obtain the equation for the outline of the generator strips. From the known profile of the strips one can determine the variation of the field in the load with allowance for diffusion into the conducting walls. Among the possible modes of operation of a generator one can single out a critical mode when the field in the load remains constant.

For the study of this mode we will assume that the pumping of the generator took place with a constant current with a duration t_0. The initial distribution of the field in the conductor is expressed by the equation

$$B_0(x) = 1 - \Phi\left(x\sqrt{\frac{\mu}{4t_0}}\right) \tag{2.8}$$

where Φ is the probability integral.

The reduced inductance satisfies the equation

$$\frac{dL}{dt} + \frac{2}{\sqrt{\pi\mu}}\frac{L}{\sqrt{t_0+t}} + \frac{1}{\tau} = 0 \tag{2.9}$$

with the initial condition $L(0) = 1$. The presence of the two conditions $L(t) = \lambda^{-1}$ and $L'(1) = -(l\lambda)^{-1}$ makes it possible to find from the known values of μ, t_0, and λ the critical load parameters l_* and τ_*, which are related to each other by the equation

$$\tau_* = \lambda\left[l_*^{-1} - \frac{2}{\sqrt{\pi\mu(1+t_0)}}\right]^{-1} \tag{2.10}$$

The ratio of the length of the load to the length of the generator is

$$\begin{aligned} l_* = \frac{\sqrt{\pi\mu}}{2}\left\{\sqrt{1+t_0} - \frac{\sqrt{\pi\mu}}{4} - \left(\sqrt{t_0} - \frac{\sqrt{\pi\mu}}{4}\right)\exp\left[\frac{4}{\sqrt{\pi\mu}}(\sqrt{t_0} - \sqrt{1+t_0})\right]\right\}\times \\ \times\left\{\left(\lambda - \frac{\sqrt{t_0} - \sqrt{\pi\mu}/4}{\sqrt{1+t_0}}\right)\exp\left[\frac{4}{\sqrt{\pi\mu}}(\sqrt{t_0} - \sqrt{1+t_0})\right] - \frac{\sqrt{\pi\mu}}{4}\frac{1}{\sqrt{1+t_0}}\right\}^{-1} \end{aligned} \tag{2.11}$$

Knowing the critical parameters l_* and τ_* in a concrete case, one can analyze the operation of the generator. If it turns out that the relaxation time τ is greater than τ_*, while $l = l_*$, then the field will increase with time. With the equality $\tau = \tau_*$ the compression of the field will be stronger for a shorter load. Descriptions of the critical modes for an inductive and for an active load are given below.

3. The case of a purely inductive load corresponds to $\tau_* \to \infty$ and $l_* = \sqrt{\pi\mu(1 + t_0)}/2$. For a load with $l > l_*$ there will be an unfavorable mode with a reduction in the field.

Using Eq. (2.5) for the reduced inductance we can find the critical profile for the strips:

$$Z_* = \frac{\sqrt{1+t_0+y}}{\sqrt{1+t_0}}\exp\left[\frac{4}{\sqrt{\pi\mu}}(\sqrt{1+t_0+y} - \sqrt{1+t_0})\right] \tag{3.1}$$

Here $-1 \le y \le 0$.

In the case of rapid pumping ($t_0 \to 0$) for ideal conductors we have $Z_{*r} = \sqrt{1 + y}$. By constructing the family of critical profiles one can select modes with an increase or decrease in the field for a given μ if the working profiles are taken lower or higher than the critical profile. In the case of slow pumping and $\mu >> 1$ in the critical mode the strips of the generator have a constant width. With a decrease in the conductance the strips will taper toward the start of the generator.

4. An active load corresponds to $l_* \to 0$ and $\lambda \to \infty$

$$\tau_* = \frac{\sqrt{\pi\mu}}{2}\left\{\left(\sqrt{1+t_0} - \frac{\sqrt{\pi\mu}}{4}\right)\exp\left[-\frac{4}{\sqrt{\pi\mu}}(\sqrt{t_0} - \sqrt{1+t_0})\right] - \left(\sqrt{t_0} - \frac{\sqrt{\pi\mu}}{4}\right)\right\} \tag{4.1}$$

For modes with an increasing field one must choose $\tau > \tau_*$. The profile of the strips in the critical mode is determined by the equation

$$\begin{aligned} Z_* = [1 + t_0 + y]^{1/2}\left\{\frac{\sqrt{\pi\mu}}{4} + \left(\sqrt{1+t_0} - \frac{\sqrt{\pi\mu}}{4}\right)\right. \\ \left.\times\exp\left[\frac{4}{\sqrt{\pi\mu}}(\sqrt{1+t_0} - \sqrt{1+t_0+y})\right]\right\}^{-1} \end{aligned} \tag{4.2}$$

In the case of slow pumping and also for ideal conductors and rapid pumping $\tau_* = 1$ we obtain the well-known result on the operation of a generator with strips of constant width at the critical resistance determined from the electrotechnical calculation of [4]. Equation (2.7) permits one to calculate the operation of a generator from the given parameters λ, μ, and t_0. Having determined the critical load at which the field remains constant one can choose the appropriate profile of the strips to obtain a mode with an increasing or decreasing field. These calculations can be performed for an arbitrary pumping mode and for any initial field distribution.

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